Tables + Concavity (1.1, 1.4) HW

1. Which of the following tables could correspond with a function that has all of the following attributes:
   - I) $f(x)$ has a global maximum value of 6
   - II) The average rate of change of $f(x)$ over the interval $[-1,2]$ is zero
   - III) The rate of change of $f(x)$ is positive for all $x \in (-\infty, 1)$
   - IV) The rate of change of $f(x)$ is negative for all $x \in (1, \infty)$

(A)

x	-3	-1	0	1	2	4	6
$f(x)$	-2	5	6	8	6	1	0

(B)

x	-3	-1	0	1	2	4	6
$f(x)$	-2	4	5	6	4	3	0

(C)

x	-3	-1	0	1	2	4	6
$f(x)$	-5	-4	0	6	-4	-3	-2

(D)

x	-3	-1	0	1	2	4	6
$f(x)$	0	5	6	6	5	1	-1

2. Tommy is removing manure from a pile on his uncle's farm. He begins at 8 AM and removes the manure at a constant rate until noon. He then stops for an hour so that he can eat lunch (after carefully washing his hands). He begins again at 1 PM and removes manure at a constant rate until the pile is empty at 4 PM. Let $M(t)$ be the volume of manure, measured in tons, left in the pile t hours after Tommy begins working. If Tommy works more slowly after lunch than before lunch, which of the following tables could depict the situation?

(A)

t	0	2	4	5	6	7	8
$M(t)$	6	4.8	3.6	3.6	2.4	1.2	0

(B)

t	0	2	4	5	6	7	8
$M(t)$	6	3.6	2.4	2.4	1.4	0.6	0

(C)

t	0	2	4	5	6	7	8
$M(t)$	6	4.2	2.4	2.4	1.6	0.8	0

(D)

t	0	2	4	5	6	7	8
$M(t)$	6	4.5	3	3	2	1	0

3. Tammy is removing manure from a pile on her aunt's farm. She begins at 8 AM and removes the manure until noon. As she works that morning, she gets more efficient, so the rate at which she removes manure speeds up throughout the morning. At noon, Tammy stops for an hour so that she can eat lunch (after carefully washing her hands). She begins again at 1 PM and removes manure until the pile is empty at 4 PM. After lunch, Tammy is tired, so the rate at which she removes manure slows throughout the afternoon. Let $M(t)$ be the volume of manure, measured in tons, left in the pile t hours after Tammy begins working. Which of the following tables could depict the situation?

(A)

t	0	2	4	5	6	8
$M(t)$	6	5.4	3	3	1.8	0

(B)

t	0	2	4	5	6	8
$M(t)$	6	4.2	3.6	3.6	2.4	0

(C)

t	0	2	4	5	6	8
$M(t)$	6	3.7	2.4	2.4	1.2	0

(D)

t	0	2	4	5	6	8
$M(t)$	6	3.6	2.8	2.8	1.2	0

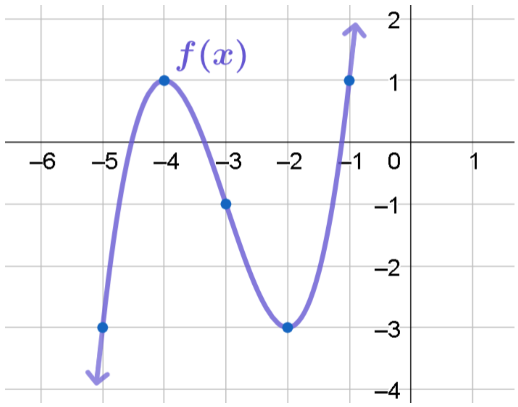
4. $f(x)$ is a function with select values given below. Decide if each of the following statements is possible. If not possible, explain why.

x	-5	0	5	10	15	20	25
$f(x)$	10	2	0	1	2	0	-8

I) $f(x)$ decreases over the interval $[-5,10]$

II) $f(x)$ is concave up on the interval $(-5,5)$

III) The rate of change of $f(x)$ decreases on the interval $(5,25)$



Consider the graph of the cubic polynomial $f(x)$ given to the left.

5. Over what interval(s) is $f(x)$ concave up?

6. Over what interval(s) is $f(x)$ concave down?

7. Give the coordinates of the point of inflection of $f(x)$.

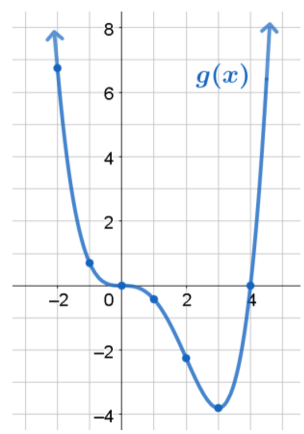
Consider the graph of the quartic polynomial $g(x)$ given to the right. ("quartic" means the degree is 4. "right" means the opposite of left.)

8. On what interval(s) is the rate of change of $g(x)$ positive?

9. On what interval(s) is the rate of change of $g(x)$ increasing?

10. On what interval(s) is the rate of change of $g(x)$ decreasing?

11. At what x-coordinate(s) does $g(x)$ have points of inflection?



12. The function f is given by $f(x) = -\frac{1}{3}x^2 - \frac{11}{4}x + 6$. Without the use of a calculator, determine which one of the following statements is true. (Hint: do you have a vague sense of the shape of this graph?)
    (A) The average rate of change over the interval $-9 \le x \le -5$ is less than the average rate of change over the interval $-5 \le x \le -1$.
    (B) The average rate of change over the interval $-9 \le x \le -5$ is greater than the average rate of change over the interval $-5 \le x \le -1$.
    (C) The average rate of change over the interval $-9 \le x \le -5$ is equal to the average rate of change over the interval $-5 \le x \le -1$.

13. If $f(x)$ increases at a decreasing rate for all x, which of the following could be a table of values for $f(x)$?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

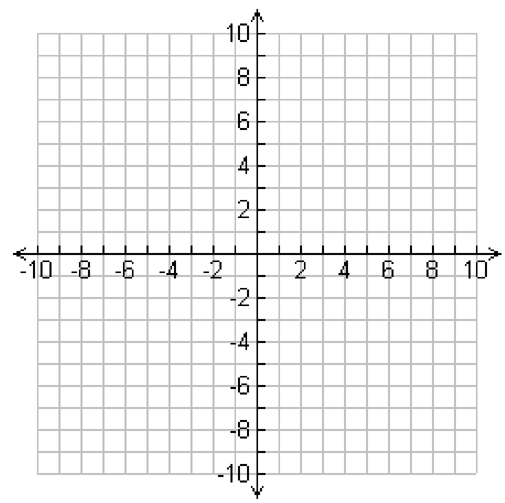
(E)

x	$f(x)$
-1	4
0	6
1	7

14. On the graph provided to the right, sketch a function $f(x)$ for which the following are true:
   - $f(2) = -3$ is a relative minimum
   - $(4,1)$ is the only point of inflection
   - $f(6) = 5$ is a relative maximum
   - f decreases at an increasing rate on the interval $(-\infty, 2)$

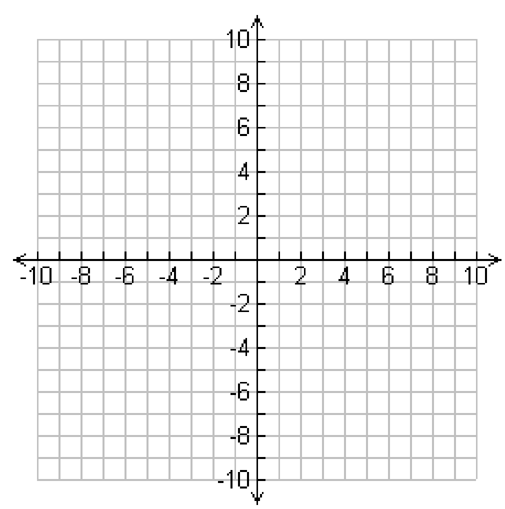
   Over what interval is $f(x)$ concave down?

   Over what interval is the rate of change of $f(x)$ positive?



15. On the graph provided to the right, sketch a function $f(x)$ for which the following are true:
   - f is decreasing and concave up on the interval $(-\infty, 0)$
   - f is increasing and concave up on the interval $(0,2)$
   - f is increasing and concave down on the interval $(2,6)$
   - f is increasing and concave up on the interval $(6, \infty)$

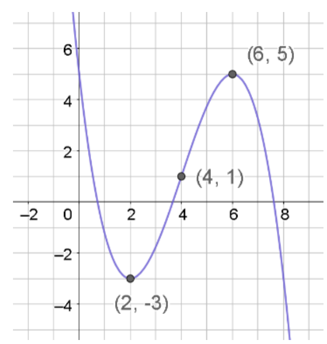
   At what x-coordinate(s) does $f(x)$ have a point of inflection?



Click/scan the QR code to access a website where you can practice vocabulary related to concavity. Make sure you can consistently earn a score of at least 60. Consistently earning a score of 120 should leave you confident you will not miss a similar question on the quiz.

Tables + Concavity (1.1, 1.4) HW Answers

1. B
2. C (Before $t = 4$, Tommy removes at a constant rate of $0.9\frac{\text{ton}}{\text{hour}}$. From $t = 4$ to $t = 5$, the rate of change is zero. After lunch, Tommy removes at a rate of $0.8\frac{\text{ton}}{\text{hour}}$)
3. A
4. Statement I: Impossible since $5 < 10$ and $f(5) < f(10)$. To be decreasing, the smaller x-value has to correspond with the greater y-value no matter which points we consider in the interval.
   Statement II: Possible
   Statement III: Impossible. Over the interval $[5,10]$, the average rate of change of $f(x)$ is $\frac{f(10)-f(5)}{10-5} = \frac{1}{5}$. Over the interval $[10,15]$, the average rate of change of $f(x)$ is $\frac{f(15)-f(10)}{15-10} = \frac{1}{5}$. Based on the average rate of change not decreasing, the rate of change of $f(x)$ cannot be decreasing.
5. $(-3, \infty)$
6. $(-\infty, -3)$
7. $f(-3) = -1$ (You can write $(-3, -1)$. It just looked weird since the answer to the other questions in this section are intervals)
8. $(3, \infty)$
9. The interval $(-\infty, 0)$ and the interval $(2, \infty)$
10. $(0,2)$
11. $x = 0$ and $x = 2$
12. (B)
    $f(x)$ is a downward opening parabola and is therefore concave down over its entire domain. This means that the rate of change (and therefore average rates of change too) is decreasing as x grows. The average rate of change over the interval $-9 \le x \le -5$ is greater than the average rate of change over the interval $-5 \le x \le -1$.
13. E
14. Answers may vary, but the most straightforward answers would be:
    $f(x)$ is concave down on $(4, \infty)$
    The rate of change of $f(x)$ is positive on $(2,6)$



15. Answers may vary, but the most straightforward answers may be:
    $f(x)$ has a point of inflection at $x = 2$ and at $x = 6$ (as these are where concavity switches)